Evolution of Spin Density Matrix in Pure NQR

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The exact time evolution of the spin density matrix for pure NQR of a spin $I = 3/2$ system under an asymmetric quadrupole Hamiltonian is given. This extends the results of a previous publication*.

In our earlier paper [1] entitled "A Spherical Tensor Method for Pure NQR", Tables 2A and 2B contain a number of errors which, if used to calculate the evolution of the density matrix for a spin 3/2 under the influence of an electric quadrupole interaction with a non-negligible asymmetry term η, will lead to incorrect results.

In this note, as in [1] the Hamiltonian

$$H_{\text{asymm}} = \frac{e^2 Q q}{4I(2I-1)} \left[(3 I_z^2 - I^2) + \frac{\eta}{2} (I_+^2 + I_-^2) \right] \quad (1)$$

and the density operator expanded in a spherical tensor operator $\mathscr{Y}^{(k)q}(I)$ basis

$$\sigma(t) = \frac{1}{2I+1} \sum_{k=0}^{2I} \sum_{q=-k}^{k} \mathscr{Y}^{(k)q}(I)\, \Phi_q^{(k)}(t) \quad (2)$$

are used. The polarizations $\Phi_q^{(k)}(t)$ are calculated from any initial conditions as

$$\Phi_q^{(k)}(t) = \sum_{k'q'} M_{qq'}^{kk'}(t)\, \Phi_{q'}^{(k')}(0)\,. \quad (3)$$

The matrix elements $M_{qq'}^{kk'}(t)$ obviate the need for Tables 2A and 2B in [1], since (3) gives a new and useful result for the full evolution in [1] of a spin 3/2 in NQR.

The M's are found by a similar procedure as outlined in [1]. Using the symmetry

$$M_{qq'}^{kk'} = (-1)^{k+k'} M_{-q\,-q'}^{kk'} = (-1)^{k+k'} M_{q'q}^{k'k}$$

Table 1.

$$M_{1\,1}^{1\,1} = \frac{2(2\eta^2+3)}{5B^2} + \frac{(\eta^2+9)}{5B^2} \cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right);$$

$$M_{1\,1}^{1\,2} = \frac{3}{\sqrt{5}\,B} \sin\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right);$$

$$M_{1\,1}^{1\,3} = \frac{\sqrt{3}}{5\sqrt{2}\,B^2} \left[(2-\eta^2) + (\eta^2-6) \cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right) \right];$$

$$M_{1\,-1}^{1\,1} = \frac{6\eta}{5B^2} \left(1 - \cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right)\right);$$

$$M_{1\,-1}^{1\,2} = \frac{\eta}{\sqrt{5}\,B} \sin\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right);$$

$$M_{1\,-1}^{1\,3} = \frac{\sqrt{6}\,\eta}{10B^2} \left(1 - \cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right)\right);$$

$$M_{1\,3}^{1\,3} = \frac{-3\eta}{\sqrt{10}\,B^2} \left(1 - \cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right)\right);$$

$$M_{1\,-3}^{1\,3} = \frac{\eta^2}{\sqrt{10}\,B^2} \left(1 - \cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right)\right);$$

$$M_{1\,1}^{2\,2} = \cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right); \qquad M_{1\,1}^{2\,3} = \frac{\sqrt{6}}{\sqrt{5}\,B} \sin\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right);$$

$$M_{1\,-1}^{2\,2} = 0; \qquad M_{1\,-1}^{2\,3} = \frac{\sqrt{3}\,\eta}{\sqrt{10}\,B} \sin\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right);$$

$$M_{1\,3}^{2\,3} = -\frac{\eta}{\sqrt{2}\,B} \sin\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right); \qquad M_{1\,-3}^{2\,3} = 0;$$

$$M_{1\,1}^{3\,3} = \frac{(7\eta^2+18)}{10B^2} + \frac{3(\eta^2+4)}{10B^2} \cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right);$$

$$M_{1\,-1}^{3\,3} = \frac{-6\eta}{5B^2} \left(1 - \cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right)\right);$$

$$M_{1\,3}^{3\,3} = \frac{\sqrt{3}\,\eta}{\sqrt{5}\,B^2} \left(1 - \cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right)\right);$$

$$M_{1\,-3}^{3\,3} = \frac{\sqrt{3}\,\eta^2}{2\sqrt{5}\,B^2} \left(1 - \cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right)\right);$$

$$M_{3\,3}^{3\,3} = \frac{(\eta^2+6)}{2B^2} + \frac{\eta^2}{2B^2} \cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right); \qquad M_{3\,-3}^{3\,3} = 0;$$

$$M_{0\,0}^{2\,3} = 0; \qquad M_{0\,0}^{1\,1} = \frac{\eta^2+15}{5B^2} + \frac{4\eta^2}{5B^2} \cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right);$$

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Table 1 (continued)

$$M^{1\,2}_{0\,2} = -\frac{\eta\sqrt{2}}{\sqrt{5}\,B}\sin\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right); \qquad M^{1\,2}_{0\,0} = 0\,;$$

$$M^{1\,3}_{0\,2} = -\frac{\sqrt{6}\,\eta}{\sqrt{5}\,B^2}\left(1-\cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right)\right);$$

$$\mathrm{M}^{1\,3}_{0\,0} = \frac{-2\,\eta^2}{5\,B^2}\left(1-\cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right)\right);$$

$$M^{2\,2}_{2\,2} = \frac{\eta^2}{2\,B^2}+\frac{(\eta^2+6)}{2\,B^2}\cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right);$$

$$M^{2\,2}_{2\,0} = \frac{\eta\sqrt{6}}{2\,B^2}\left(1-\cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right)\right);$$

$$M^{2\,\,2}_{2\,-2} = \frac{\eta^2}{2\,B^2}\left(1-\cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right)\right);$$

$$M^{2\,3}_{2\,2} = \frac{\sqrt{3}}{B}\sin\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right);$$

$$M^{2\,3}_{2\,0} = \frac{\eta}{B\sqrt{10}}\sin\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right); \qquad M^{2\,\,3}_{2\,-2} = 0\,;$$

$$M^{2\,2}_{0\,0} = \frac{3}{B^2}+\frac{\eta^2}{B^2}\cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right);$$

$$M^{2\,3}_{0\,2} = \frac{-\eta}{\sqrt{2}\,B}\sin\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right);$$

$$M^{3\,3}_{2\,2} = \frac{\eta^2}{2\,B^2}+\frac{\eta^2+6}{2\,B^2}\cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right);$$

$$M^{3\,3}_{2\,0} = \frac{-\eta\sqrt{3}}{\sqrt{10}\,B^2}\left(1-\cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right)\right);$$

$$M^{3\,\,3}_{2\,-2} = \frac{\eta^2}{2\,B^2}\left(1-\cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right)\right);$$

$$M^{3\,3}_{0\,0} = \frac{4\,\eta^2+15}{5\,B^2}+\frac{\eta^2}{5\,B^2}\cos\left(\frac{B\tilde{Q}t}{\sqrt{3}}\right);$$

Everywhere above $B^2=(\eta^2+3)$.

the results are presented in Table 1. The sum over k' and q' in (3) is restricted by Eqs. (14) and (16) in [1].

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[1] M. S. Krishnan and B. C. Sanctuary, Z. Naturforsch. **41 a,** 353 (1986).

Errata

F. Winterberg, Relativistic Electron-Positron Gamma Ray Laser, Z. Naturforsch. **41 a,** 1005–1008 (1986).

Due to a misprint in the typewritten manuscript the following corrections have to be made:

1. Equation (3) must read $v_\perp^2/c^2 \cong I/\gamma I_A$ and Eq. (3a) $v_\perp^2/c^2 \cong (I/\gamma I_A)\sqrt{1-v_\perp^2/c^2}$.

F. Winterberg, A Crucial Test for Einstein's Special Theory of Relativity Against the Lorentz-Poincaré Ether Theory of Relativity, Z. Naturforsch. **41 a,** 1261–1266 (1986).

The following corrections have to be made:

1. At the end of the sentence following Eq. (1) it must read $l/r > \sqrt{E/\sigma} \cong 10$.
2. Equation (2) must read $\operatorname{tg}\psi = \gamma\operatorname{tg}\varphi$.
3. In Eq. (10) replace ω_2 with ω_1.
4. Equation (8) must read $\alpha(t) = A\sin(2\omega t + 2\delta)$.
5. In Eq. (18) replace $\sin(2\omega t - \pi/2)$ with $\sin(2\omega t - \pi)$.

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